A simple model for multiple equilibria in ice-covered oceans

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Problem formulation

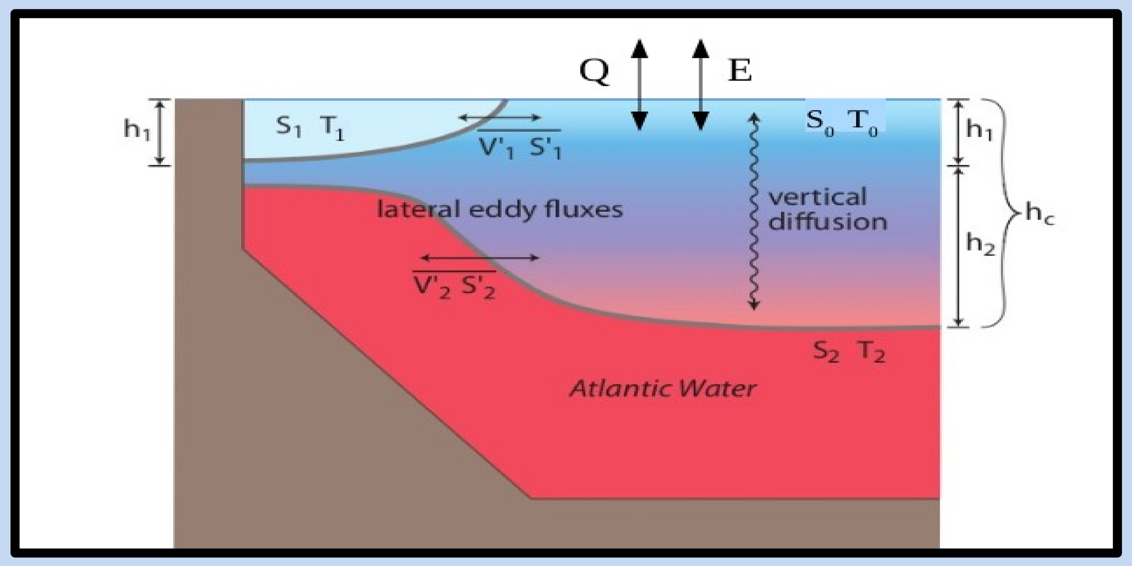
The Arctic Ocean is dominated by a cold, fresh halocline in the upper 50-200 m and a warm, salty water mass of Atlantic origin below. The halocline shields the overlying sea ice from being melted by the warm Atlantic Water

The surface albedo is small for open water and large for ice/snow cover, allowing for possible feedbacks between the surface heat budget and ice cover. (ice-albedo feedback)

Diapycnal mixing in the Arctic is small, in part due to the presence of sea ice. It is thought that mixing would be larger if the Arctic were ice free. (wind-ice-ocean feedback)

We seek guidance on what determines whether multiple equilibria can exist

A conceptual model for the Arctic Ocean



The salinity and temperature in the basin interior are determined by a balance between lateral eddy fluxes, diapycnal mixing, and air-sea exchange

Layers 1+2 $$Ph_2\overline{V_2'S_2'}+Ph_1\overline{V_1'S_1'}=-AES_R$$

Layer 2 $$Ph_2\overline{V_2'S_2'}=\frac{KA(S_2-S_0)}{h_1+h_2}$$

P =perimeter, A=area, K=diapycnal mixing
E=evaporation – precipitation

The eddy flux is parameterized as

$$\overline{V_k'S_k'}=c_kV_k(S_k-S_0) \qquad V_k=\frac{g\alpha_S(S_k-S_0)h_k}{\rho_0 f_0 L}$$

Nondimensionalize variables:

$$\delta S_1=\frac{S_0-S_1}{S_2-S_1} \qquad \delta S_2=\frac{S_2-S_0}{S_2-S_1}$$

$$\delta T_1=\frac{\alpha_T(T_0-T_1)}{\alpha_S(T_2-T_1)} \qquad \delta T_2=\frac{\alpha_T(T_2-T_0)}{\alpha_S(T_2-T_1)}$$

Analytic solutions

We can derive equations for the interior salinity δS_1 and h_2

$$(\delta S_1^2-\phi)^{3/2}-\lambda(1-\delta S_1)^2=0 \qquad h_2=(c_1/c_2)^{0.5}\frac{(\delta S_1^2-\phi)^{1/2}}{1-\delta S_1}$$

Solve for temperature subject to a surface heat flux

$$Q=G(T_0-T_A)+(1-\alpha)Q_S$$

T_A is atmospheric temperature
G is restoring constant (W/m^2 C)
α = albedo (0.6 ice cover, 0.1 open ocean)
Q_S is solar radiation

We can also derive a solution for the temperature of the halocline

$$\delta T_2=\frac{\gamma\delta T_A+\delta S_1\delta T_B-(1-\alpha)Q_S}{\gamma+\delta S_1+c_1/c_2 h_2^2(1-\delta S_1)}$$

Nondimensional parameters

$$\phi=\frac{AES_R}{c_1\psi_g(S_2-S_1)}$$ evap minus precip

$$\gamma=\frac{AG}{c_1\rho_0 C_p\psi_g}$$ cooling

$$\lambda=\frac{c_2^{0.5}KA}{c_1^{1.5}\psi_g h_1}$$ mixing

$$\psi_g=\frac{gP\alpha_S(S_2-S_1)}{\rho_0 f_0 L}$$ reference transport

$$Q_S=\frac{\alpha_T AQ}{c_1\rho_0 C_p\alpha_S(S_2-S_1)}$$ solar radiation

$$\delta T_B=\frac{\alpha_T(T_2-T_1)}{\alpha_S(S_2-S_1)}$$ boundary temperature

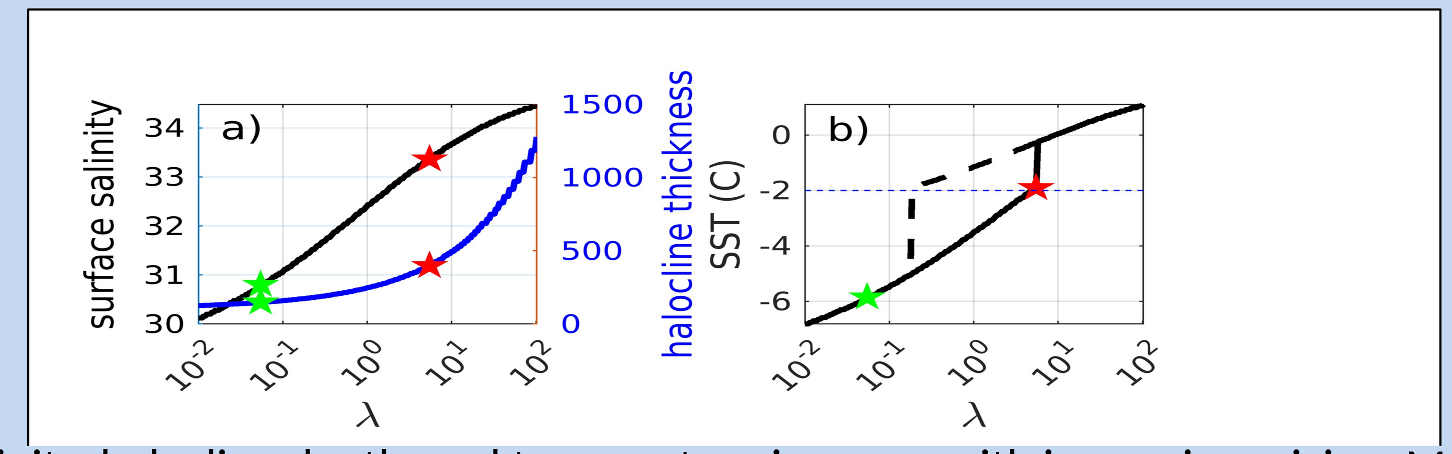
$$\delta T_A=\frac{\alpha_T(T_2-T_A)}{\alpha_S(S_2-S_1)}$$ atmospheric temperature

For typical parameters:

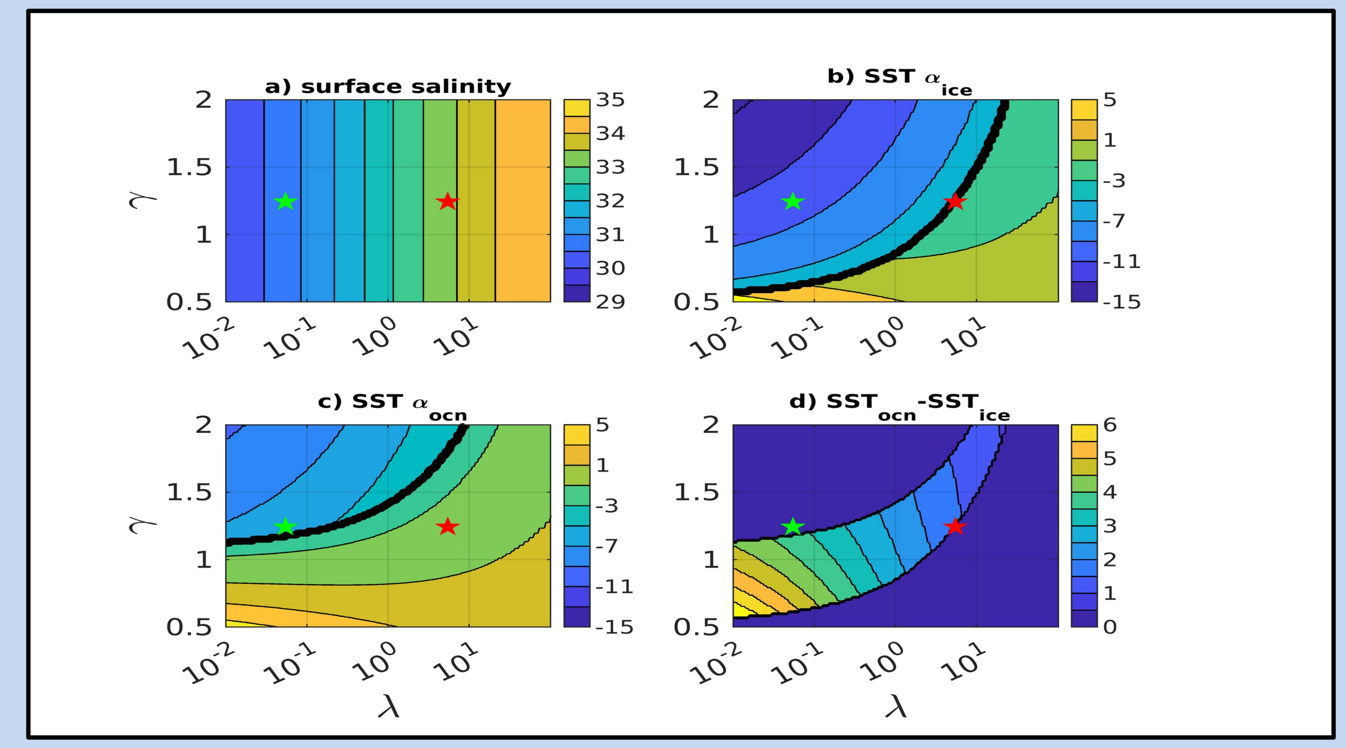
$\lambda \ll 1$ weak mixing	$\lambda \gg 1$ strong mixing
$\gamma=O(1)$ cooling	$\phi=O(10^{-2})$ precipitation
$Q_S=O(10^{-1})$ solar radiation	$\delta T_A=O(10^{-1})$ atmospheric temp

Results

Dependence of halocline on mixing



Salinity, halocline depth, and temperature increase with increasing mixing. Multiple equilibria are possible, one state with ice-covered with high albedo and one ice-free with low albedo (ice-albedo feedback)



ME exist over a finite range of cooling, most prevalent for weak mixing
Also get ME if mixing gets large when the ice melts (wind-ice-ocean feedback)

Similar analysis can be done for all forcing parameters...

SUMMARY

Coupled equations for the temperature, salinity, and halocline depth have been derived that describe a balance between lateral eddy fluxes, vertical mixing, and air-sea exchange

The system supports multiple equilibria resulting from:

> The dependence of albedo on ice cover

> The expected increase in mixing as the Arctic becomes ice-free

> Asymptotic solutions give explicit expressions for weak and strong mixing regimes (ice-covered and ice-free), including the level of mixing required to melt the ice

> Existence of ice cover depends on many parameters
External forcing
(atmospheric temperature, E-P, cooling rate, solar, source waters)
Environmental parameters of the basin
(e.g. size, shelf depth, shelf width, mixing rate)